AP - Probabilité, inégalités de Bell & intrication quantique

You have nothing to do but to mention the quantum theory, and people will take your voice for the voice of science, and believe anything Bernard Shaw - Geneva (1938)

Source de cette activité : cours *Quantum Computation* de John Preskill (professeur à Caltech, `http://www.theory.caltech.edu/people/preskill/ph229/`)

1 Mise en place du problème

Question 1.1. *Lire le texte suivant. Trouver la traduction des mots dans la colonne de droite. De quoi le texte parle-t-il ? En quoi cet histoire peut-elle avoir un lien avec des mathématiques ?*

The system that Alice and Bob are studying might be described this way : Alice, in Pasadena, has in her possession three coins laid out on a table, labeled 1,2,3. Each coin has either its heads (H) or tails (T) side facing up, but it is hidden under an opaque cover, so that Alice is not able to tell whether it is an H or a T. Alice can uncover any one of the three coins, and so learn its value (H or T). However, as soon as that one coin is uncovered, the other two covered coins instantly disappear in a puff of smoke, and Alice never gets an opportunity to uncover the other coins. She has many copies of the three-coin set, and eventually she learns that, no matter which coin she exposes, she is just as likely to find an H as a T. Bob, in Chicago, has a similar set of coins, also labeled 1,2,3. He too finds that each one of his coins, when revealed, is as likely to be an H as a T. In fact, Alice and Bob have many identical copies of their shared set of coins, so they conduct an extensive series of experiments to investigate how their coin sets are correlated with one another. They quickly make a remarkable discovery : Whenever Alice and Bob uncover coins with the same label (whether 1, 2, or 3), they always find coins with the same value — either both are H or both are T. They conduct a million trials, just to be sure, and it works every single time! Their coin sets are perfectly correlated.	**Vocabulary** Pasadena laid out on a table heads tails hidden uncover puff of smoke is as likely to be extensive series of experiments investigate trial set

Question 1.2. *Alice retourne la pièce numéro 1 et trouve H (face). Que va trouver Bob s'il retourne lui aussi la pièce 1 ? Bob a-t-il intérêt à retourner cette pièce, plutôt qu'une autre ?*

2 Désenchantement d'Alice

Question 2.1. *Lire le dialogue suivant. De quoi Alice et Bob parlent-ils ? Quel problème expérimental remarquent-ils ? Que vont-ils faire ?*

Alice and Bob suspect that they have discovered something important, and they frequently talk on the phone to brainstorm about the implications of their results. One day, Alice is in an especially reective mood :

Alice: You know, Bob, sometimes it's hard for me to decide which of the three coins to uncover. I know that if I uncover coin 1, say, then coins 2 and 3 will disappear, and I'll never have a chance to find out the values of those coins. Once, just once, I'd love to be able to uncover two of the three coins, and find out whether each is an H or a T. But I've tried and it just isn't possible — there's no way to look at one coin and prevent the other from going poof!

Bob: [Long pause] Hey ... wait a minute Alice, I've got an idea ... Look, I think there is a way for you to find the value of two of your coins, after all! Let's say you would like to uncover coin 1 and coin 2. Well, I'll uncover my coin 2 here in Chicago, and I'll call you to tell you what I found, let's say its an H. We know, then, that you are certain to find an H if you uncover your coin 2 also. There's absolutely no doubt about that, because we've checked it a million times. Right?

Alice: Right ...

Bob: But now there's no reason for you to uncover your coin 2; you know what you'll find anyway. You can uncover coin 1 instead. And then you'll know the value of both coins.

Alice: Hmmm ... yeah, maybe. But we won't be sure, will we? I mean, yes, it always worked when we uncovered the same coin before, but this time you uncovered your coin 2, and your coins 1 and 3 disappeared, and I uncovered my coin 1, and my coins 2 and 3 disappeared. There's no way we'll ever be able to check anymore what would have happened if we had both uncovered coin 2.

Bob: We don't have to check that anymore, Alice; we've already checked it a million times. Look, your coins are in Pasadena and mine are in Chicago. Clearly, there's just no way that my decision to uncover my coin 2 can have any influence on what you'll find. It's just that when I uncover my coin 2 we're collecting the information we need to predict with certainty what will happen when you uncover your coin 2. Since we're already certain about it, why bother to do it!

Alice: Okay, Bob, I see what you mean. Why don't we do an experiment to see what really happens when you and I uncover different coins?

Bob: I don't know, Alice. We're not likely to get any funding to do such a dopey experiment. I mean, does anybody really care what happens when I uncover coin 2 and you uncover coin 1?

Alice: I'm not sure, Bob. But I've heard about a theorist named Bell. They say that he has some interesting ideas about the coins. He might have a theory that makes a prediction about what we'll find. Maybe we should talk to him.

Bob: Good idea! And it doesn't really matter whether his theory makes any sense or not. We can still propose an experiment to test his prediction, and they'll probably fund us.

3 Entracte

Question 3.1. *Lire la suite du texte. Qu'est ce que le CERN, et qui est ce fameux Bell ?*

So Alice and Bob travel to CERN to have a chat with Bell. They tell Bell about the experiment they propose to do. Bell listens closely, but for a long time he remains silent, with a faraway look in his eyes. Alice and Bob are not bothered by his silence, as they rarely understand anything that theorists say anyway. But finally Bell speaks.

Bell: I think I have an idea .... When Bob uncovers his coin in Chicago, that can't exert any influence on Alice's coin in Pasadena. Instead, what Bob finds out by uncovering his coin reveals some information about what will happen when Alice uncovers her coin.

Bob: Well, that's what I've been saying ...

Bell: Right. Sounds reasonable. So let's assume that Bob is right about that. Now Bob can uncover any one of his coins, and know for sure what Alice will find when she uncovers the corresponding coin. He isn't disturbing her coin in any way; he's just finding out about it. We're forced to conclude that there must be some hidden variables that specify the condition of Alice's coins. And if those variables are completely known, then the value of each of Alice's coins can be unambiguously predicted.

Bob: [Impatient with all this abstract stuff] Yeah, but so what?

Lorsqu'il parle des variables cachées, Bell fait les deux hypothèses suivantes :

- Il existe des variables cachées, dans le sens où l'on ne les connait pas car elles prennent des valeurs aléatoire
- Mais, si on connaissait ces variables, alors le résultat de n'importe quel expérience pourrait être prédit à l'avance
- On suppose que la décision que prend Bell à Chicago (retourner la pièce numéro 1) a aucun effet sur les variables cachées d'Alice à Pasadena. (On dit que les variables cachées sont locales).

Question 3.2. *Ces hypothèses vous apparaissent-elles acceptables ?*

4 La théorie de Bell : fameuse ou fumeuse ?

La suite est un long monologue de Bell. L'objectif est, sous l'hypothèse des variables cachées, d'arriver à une inégalité. Ensuite, Alice et Bob feront les mesures pour voir si cette inégalité est vérifiée expérimentalement ou non. Si elle ne l'est pas, alors l'hypothèse des variables cachées sera fausse ! Lire le texte et répondre aux questions *au fur et à mesure.*

Bell: When your correlated coin sets are prepared, the values of the hidden variables are not completely specified; that's why any one coin is as likely to be an H as a T. But there must be some probability distribution P(x,y,z) (with $x, y, z \in \{H, T\}$) that characterizes the preparation and governs Alice's three coins. These probabilities must be nonnegative, and they sum to one:

$$\sum_{x,y,z} P(x, y, z) = 1 \tag{1}$$

Question 4.1. *Que représente P(HHH) ? P(HTH) ? Combien de combinaison possible y-a-t-il ? (par exemple HHH et HTH sont deux combinaisons)*

Question 4.2. *Expliquer la phrase "These probabilities must be nonnegative, and they sum to one"*

Alice can't uncover all three of her coins, so she can't measure P(x,y,z) directly. But with Bob's help, she can in effect uncover any two coins of her choice.

Let's denote with $P_{same}(i, j)$, the probability that coins i and j (i,j = 1,2,3) have the same value, either both H or both T. Then we see that

$$P_{same}(1, 2) = P(HHH) + P(HHT) + P(TTH) + P(TTT) \tag{2}$$

$$P_{same}(2, 3) = P(HHH) + P(THH) + P(HTT) + P(TTT) \tag{3}$$

$$P_{same}(1, 3) = P(HHH) + P(HTH) + P(THT) + P(TTT) \tag{4}$$

and it immediately follows from eq. 1 that

$$P_{same}(1, 2) + P_{same}(2, 3) + P_{same}(1, 3) = 1 + 2P(HHH) + 2P(TTT) \geq 1 \tag{5}$$

So that's my prediction: P_{same} should obey the inequality

$$P_{same}(1, 2) + P_{same}(2, 3) + P_{same}(1, 3) \geq 1 \tag{6}$$

You can test it my doing your experiment that "uncovers" two coins at a time.

Bob: Well, I guess the math looks right. But I don't really get it. Why does it work?

Alice: I think I see .... Bell is saying that if there are three coins on a table, and each one is either an H or a T, then at least two of the three have to be the same, either both H or both T. Isn't that it, Bell?

Bell stares at Alice, a surprised look on his face. His eyes glaze, and for a long time he is speechless. Finally, he speaks:

Bell: Yes.

So Alice and Bob are amazed and delighted to find that Bell is that rarest of beasts — a theorist who makes sense. With Bell's help, their proposal is approved and they do the experiment, only to obtain a shocking result. After many careful trials, they conclude, to very good statistical accuracy that

$$Psame(1, 2) \approx Psame(2, 3) \approx Psame(1, 3) \approx \frac{1}{4} \tag{7}$$

and hence

$$Psame(1, 2) + Psame(2, 3) + Psame(1, 3) \approx 3 \times \frac{1}{4} < 1 \tag{8}$$

The correlations found by Alice and Bob flagrantly violate Bell's inequality! Alice and Bob are good experimenters, but dare not publish so disturbing a result unless they can find a plausible theoretical interpretation. Finally, they become so desperate that they visit the library to see if quantum mechanics can offer any solace ...

Question 4.3. *Expliquer la phrase "The correlations found by Alice and Bob flagrantly violate Bell's inequality!"*

5 Pour aller plus loin

Question 5.1. *Quelle équipe de chercheur a réussit en premier à montrer que les inégalités de Bell pouvaient être violées ?*

Question 5.2. *Chercher ce qu'est le paradoxe EPR. En quoi cela a un lien avec les inégalités de Bell ?*